

# **EXHIBIT 15**

# Estimating Candidate Support: Comparing Iterative EI & EI-RxC Methods

IMMEDIATE

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### **Abstract**

Scholars of race and politics are concerned with estimating individual-level voting behavior from aggregate-level data. The most commonly used technique, King's Ecological Inference (EI), has been criticized for inflexibility in multiethnic settings, or with multiple candidates. One method for estimating vote support for multiple candidates in the same election is called ecological inference: row by columns (RxC). While some simulations show that RxC estimates are more accurate than the iterative EI technique, there has not been a side-by-side comparison of the two methods using real election data. We assess the two methods by comparing iterative EI and RxC models in a variety of RxC combinations including two candidates and two groups, three candidates and three groups, up to 12 candidates and three groups, and multiple candidates and four groups. Both methods produce similar results pointing to the presence of racially polarized voting, and very little differences emerge across the estimates.

## Introduction

A recurring theme for American politics scholars is the study of racially polarized voting. Since V.O. Key's seminal study of Southern politics (Key, 1949), it has been well-documented that African-Americans, Latinos, and Whites often have very different preferences and voting patterns (Barreto et al., 2005; Grofman and Migalski, 1988; Issacharoff, 1992; McCrary, 1990). Indeed, a major reason for the Voting Rights Act of 1965 was to increase the voter registration and representation of African-Americans who were being blocked from political incorporation by whites across the South. As the VRA took hold and groups sued for equal representation, courts asked social scientists to present evidence of voting patterns by race. The basic question was simple: do whites block-vote against African-American candidates and prevent African-Americans from gaining political representation? Using basic bivariate regression developed by Goodman (1953, 1959), the early evidence presented at trial supported what V.O. Key had already found. Over the decades racial demographics and social science tools have evolved considerably. No longer facing a strictly black-white hyper-segregated environment, social scientists, notably King (1997) and Grofman (1992, 1995) argued for a more precise measurement of racial voting patterns to account for an increase in racially heterogeneous neighborhoods and the rapid emergence of Latinos and Asians.

Today, social scientists—and overwhelmingly the courts—rely on two statistical approaches to ecological data. The first, ecological inference (EI), developed by King (1997) is said to be preferred when there are only two racial

or ethnic groups, and ideally only two candidates contesting office. The second, ecological inference  $R \times C$  (RxC) developed by Rosen et al. (2001), is said to be preferred when there are multiple racial or ethnic groups, or multiple candidates contesting office. While both techniques make strong theoretical cases for their approach, it is not clear that when faced with the exact same dataset, they would produce different results. In one case, analysis of the same dataset across multiple ecological approaches found they tend to produce the same conclusion (Grofman and Barreto, 2009). However, others have argued that using King's iterative EI technique with multiple racial groups or multiple candidates will fail and should not be relied on (Ferree, 2004). Still others have gone further and asserted that the iterative EI approach cannot be used to analyze multiple racial group or multiple candidate elections because "...it biases the analysis for finding racially polarized voting" (Katz, 2014).

As with any methodological advancement, there is a healthy and rigorous debate in the literature. However, very little real election data has been brought to bear in this debate. Ferree (2004) assessed King's iterative approach with simulated data and a parliamentary election in South Africa using a proportional representation system. Grofman and Barreto (2009) compared an exit poll to precinct election data in Los Angeles, but only compared Goodman's ecological regression against King's EI without evaluating the RxC approach. To address these shortcomings, we weigh in with a comprehensive analysis of real ecological voting data from 14 elections and 78 candidates in multiethnic settings across the United States.

Using real-world ecological voting data, we attempt to answer three fun-

damental questions between the iterative EI technique and the RxC method:

- 1) Does EI over-estimate racially polarized voting (RPV) compared to RxC? In other words, does EI bias towards detecting RPV?
- 2) Are there systematic outcome differences between EI and RxC when analyzing elections with few candidates versus elections with multiple candidates?
- 3) Are there systematic outcome differences between EI and RxC when analyzing elections with more than two racial groups?

With regards to the latter two questions, if RxC is indeed a “better” measure of group voting behavior in a multi-candidate context, then we should expect to see noticeably different estimates across the two methods. Specifically, relative to RxC, the iterative EI method should become unstable and possibly generate ostensibly invalid estimates in scenarios with multiple candidates and/or multiple racial/ethnic groups. Our analysis does not suggest this to be the case. We find very strong patterns of consistency across iterative EI and RxC despite claims to the contrary. Across the 78 candidates we analyzed there is no evidence that either EI or RxC are biased towards or against findings of polarized voting. Instead, we find that both methods result in the same conclusion of racial voting patterns. Further, the point estimates that both methods produce are remarkably similar, typically within 2 points of one another. For social scientists and legal scholars interested in adjudicating whether or not racially polarized voting exists, both approaches prove valuable and entirely consistent across the 14 elections and 78 candidates we analyzed.

In the pages that follow we first review the relevant literature on ecological inference as it pertains to racially polarized voting analysis. Second, we describe our myriad datasets gathered in several states spanning more

than a decade. These datasets all contain elections in areas with relatively high Latino (and Anglo) voting populations and with at least one Spanish-surnamed candidate. In addition to Latinos, many of the datasets include sizable African-American and Asian-American populations, which allows us examine how iterative EI and RxC operate in different racial and ethnic contexts. We also examine elections with two, three, four, and up to 12 different candidates, to fully assess how both models works in different environments. Beyond this, we briefly demonstrate that both the iterative EI technique and the RxC method produce results in line with individual-level data (exit polls). We then present a congruence analysis based first off of a simple 2x2 comparison, with an extension to multiple groups and candidates, to highlight the ways in which analysts can determine whether the two methods result in same substantive conclusions. Finally, we conclude with a discussion of our findings and the implications for future research and work in the area of ecological inference and racially polarized voting.

## **Ecological Inference & Racially Polarized Voting Analysis**

The challenges surrounding ecological inference are well-documented in the social science literature. Robinson (1950) pointed out that relying on aggregate data to infer the behavior of individuals can result in the ecological fallacy. Since then scholars have applied different methods to discern more accurately micro-level relationships from aggregate data. Goodman (1953, 1959) advanced the idea of ecological regression where individual patterns

can be drawn from ecological data under certain conditions. However Goodman's statistical approach assumes that group patterns are consistent across each ecological unit, and in reality that may not be the case.

Eventually, systematic analysis revealed that early methods could produce unreliable results (see e.g., King (1997)).<sup>1</sup> Ecological inference is King's (1997) solution to the ecological fallacy problem inherent in aggregate data, and since the late 1990s has been the benchmark method courts use to evaluate racially polarized voting patterns in voting rights lawsuits. King's EI has also been widely used in comparative politics research on group and ethnic voting patterns.

Some critics, however, claim that King's EI model was designed primarily for binary data (2x2) such as situations in which just two groups (e.g., blacks and whites; Hispanics and Anglos, etc.) exist. While many geographic areas (e.g., Mississippi, Alabama) still contain essentially two groups, the growth of racial groups such as Latinos and Asians have challenged the historical biracial focus on race in the U.S. To account such complexities, Rosen et al. (2001) developed a hierarchical rows by columns (RxC) approach, which can be used to analyze multiple racial groups and multiple candidates together. However, due to the computationally intensive nature of their model, this approach was not initially employed in the social sciences, in general, and

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<sup>1</sup>However, in an extensive review, Owen and Grofman (1997) concluded that despite some valid theoretical concerns, the single-equation ecological regression still holds up and provides meaningful and accurate estimates of racially polarized voting. A decade later, Grofman and Barreto (2009) evaluated how ecological models compare to one another using a combination of simulated data, actual election precinct data, and an accompanying exit poll. Their analysis demonstrated that there is general consistency across the single and double equation methods, and that once voter turnout rates are accounted for similar conclusions are reached.



in voting rights cases in specific. In addition to this, King also suggested that his method can still be used with more complex data (e.g., 3x2) by “iteratively” applying the model to different subsets of the data. In trying to assess voting patterns for three racial groups (whites, blacks and Hispanics), the iterative technique would estimate three separate equations. First, white and black turnout in a given electoral jurisdiction would be collapsed into a single category to estimate Hispanic vote choice for X candidate. Then, whites and Hispanics are grouped together to estimate X candidate support for blacks. And finally, Hispanics and blacks are collapsed into a single group versus whites to estimate X candidate support for white voters.

While this iterative EI technique has been widely used in voting rights cases, some social scientists have expressed concern. Ferree (2004), for instance, has argued that combining blacks and whites into a single “non-Hispanic” category in order to estimate Hispanic turnout can vastly overestimate Hispanic turnout due to issues of aggregation bias and multimodality in the data. This suggests that the iterative EI approach could increase the likelihood of detecting racially polarized voting due to a larger-than-reality share of Hispanics in the data. While Ferree (2004) suggested some quick “fixes”—such as accounting for the relative size of each group or changing the order in which cells are estimated—to reduce aggregation bias and multimodality caused by collapsing rows or columns, she recommended estimating the cells of the rows by columns table simultaneously rather than iteratively.<sup>2</sup> Others, such as Herron and Shotts (2003a,b), have criticized EI estimates when used for second-stage regression, given that error is baked into the

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<sup>2</sup>The simultaneous method recommended is Rosen et al.’s 2001 RxC method.

second-level regression estimation.<sup>3</sup> Some have gone even further in arguing that analysis that relies on King’s iterative approach can be “problematic and no valid statistical inferences can be drawn,” and that only the hierarchical RxC approach developed by Rosen et al. (2001) can produce reliable results in multi-ethnic and multi-candidate settings (Katz, 2014).<sup>4</sup> In explaining the theoretical reasons of why the iterative EI technique is “ill-equipped” to handle complex datasets, Katz stated that “...adding additional groups and vote choices to King’s (1997) EI is not straightforward,” and that “...given the estimation uncertainty, it may not be possible to infer which candidate is preferred by members of the group.” The argument against King’s iterative EI in the case of multiple racial group, or especially multiple candidate elections, is that EI pits candidate A versus all others who are not candidate A. If the election features four candidates (A, B, C, D), critics suggest that EI cannot accurately estimate vote choice quantities because vote for candidate A is compared against the combined vote for B, C, and D. Since the iterative approach would have to run four separate equations to obtain vote estimates for each candidate, social scientists such as Katz (2014) have even claimed that EI biases the findings in favor of bloc-voting: “...this jerry rigged approach to dealing with more than two vote choices stacks the deck in favor of finding statistical evidence for racially polarized.”

Due to these concerns, advancements in computing power, and the avail-

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<sup>3</sup>In response to this issue, Adolph and King (2003) adjusted the EI procedure to reduce inconsistencies when estimating second-stage regressions.

<sup>4</sup>Greiner and Quinn (2010) combined RxC with individual-level exit poll data, and showed that a hybrid model is perhaps even more preferable than a straight aggregation model. However, using exit poll data is not always available to researchers and practitioners. Indeed, in most county or city elections, exit poll data do not exist, which is why scholars often attempt to infer voting patterns through aggregate data.

ability of numerous packages developed for R, the computationally intensive RxC approach is now increasingly being recommended in place of the iterative EI solution. However, no study has empirically examined how these approaches perform side-by-side with real election data containing a number of different candidate and racial group combinations. Previous work has mostly leveraged Monte Carlo simulation or a select few election datasets to highlight some of the potential pitfalls of the iterative EI approach. Since we lack more expansive efforts to compare the two approaches, there simply is not enough information to enable researchers and legal practitioners to evaluate under which conditions the RxC method is more suitable or appropriate than the iterative EI technique. For example, if there are three racial groups in equal thirds of the electorate, does aggregation bias create more error in the iterative EI than a scenario in which two dominant groups comprise 90% and a small group just 10% of the electorate? Likewise, is EI's iterative approach (e.g., black vs. non-black, white vs. non-white, Hispanic vs. non-Hispanic) to candidates more stable when analyzing three candidates and far less stable when eight candidates contest the election? Is it really the case that the iterative approach is more likely than the RxC method to produce findings in favor of racially polarized voting patterns? The analytical task of this paper is to consider these questions empirically; to systematically assess whether using the iterative EI method, as opposed to the hierarchical RxC method, can change the substantive conclusions one draws as it pertains to racially polarized voting patterns. Since we take advantage of real-world election datasets of varying electoral units and sizes, candidates, and racial/ethnic groups, our study provides the most comprehensive attempt to answer some

of the preceding questions.

## Data and Methods

We turn to precinct voting data from three diverse states—California, Texas and Florida—across 14 different elections from 2004 to 2012, in which a total of 78 candidates were on the ballot, to examine how the two different methods process the same datasets. For each of the 14 elections we analyze, we have precinct-level data on candidate vote distribution, as well as the racial demographics of the voting population in each precinct, and the total numbers of ballots cast. In two states, California and Florida, we have data on the actual voters by race and ethnicity. In Texas, we have the number of eligible voters by race and ethnicity. Thus, the key variables are percent [candidate] and percent racial/ethnic group, and our estimates control for the number of total voters per precinct, as instructed by King (1997), Ferree (2004), and Rosen and colleagues (2001).

The data we examine is diverse across almost any dimension. We have data on more than 4,900 precincts in Los Angeles County or only 38 precincts in one school board district in central Florida. The elections we examine also have varying number of candidates: from a head-to-head matchup with only two candidates to elections with up to twelve candidates. The data are also diverse with respect to the number of racial or ethnic groups within the electorate, starting with jurisdictions that are primarily Latino-White, then areas with sizable Latino, White and Asian voting populations, and other geographies such as elections with Latino, White, Asian and Black voting populations. Thus, the data we bring to bear is comprehensive and

diverse across almost any metric, enabling us to follow a pattern of increasing complexity.

We begin the analysis with a basic dataset with just two candidates and just two racial groups, and then stick with these two racial groups and add election contests with three, four, five, six, seven, nine and twelve candidates. In each election we analyze, there is at least one co-ethnic candidate allowing us to assess racially polarized voting. After comparing Iterative EI and RxC results with two racial groups and multiple candidates, we next turn to analysis of multiple racial groups. We first assess only two candidates, but in two different environments with Latino, White and Asian, and then Latino, White and Black. Then we look at both multiethnic scenarios and contests with more than two candidates. Finally, we assess a very diverse electoral environment to really put the two methods to the test. We conclude with an analysis of a Democratic primary in Los Angeles County that featured seven candidates including viable Latino, White, Black and Asian candidates, and provide results for all four racial groups of voters.

[INSERT TABLE 1 ABOUT HERE]

## How “Informative” is the Data?

Before we proceed with the comparison of iterative EI and EI RxC model results, it is important to report the extent to which our datasets are amenable to ecological inference. This is crucial because some aggregate data are more “informative” about the micro-data than others (see Tam Cho and Gaines (2004)). To gauge the level of information contained in the precinct-level

datasets, we rely on tomography plots. There are two diagnostic uses for tomography plots. By plotting all the logically possible pairs of parameter values—that is, the known information—tomography lines succinctly display how constrained the parameters are and thus, how easy or difficult the estimation problem will be. In a given plot, there is one tomography line bound with the  $[0,1]$  interval for each observation. Lines that do not extend across the entire unit square are further bounded than those that cross the entire unit square. If the lines are more bounded, one may be more successful when estimating the true parameter values when relying on King’s EI approach.

In addition to showing all the available deterministic information in a problem, tomography plots also help assess whether the underlying truncated bivariate normal (TBVN) distribution imposed by King’s EI is reasonable. King asserts that an “*informative*” tomography plot can reasonably be assumed to have been generated by a truncated bivariate normal distribution (King 1997). That is, if most of the tomography lines seem to intersect in a region, it means that the actual individual-level data are most likely, but not certainly, clustered there, marking a potential location for the mode of the joint distribution of  $\beta$ ’s. If, however, no area of intersection is evident and the parameter bounds are too wide, the implication is that the TBVN distributional assumption imposed by King’s EI may not be reasonable. Stated differently, if the tomography plot is “*uninformative*,” the data is less likely to have been generated from a TBVN distribution, resulting in standard errors that may be too large to be useful or simply incorrect (King 1997, chapter 16).

When using a tomography plot to determine the suitability of using EI

for a given data, it is important to recognize that the information obtained from this diagnostic plot is only suggestive. A tomography plot does not allow a researcher to make definitive claims about the particular distributional assumptions of the data. As Tam Cho and Gaines (2004) have stated, “...deciding whether a tomography plot is informative is something of an art, no one has devised a concrete measure for ‘*informativeness*’ or any formal test for accepting or rejecting the TBVN distributional assumption (or any other distributional assumption) on the basis of the plot” (pg. 155). What this means is that tomography plots only provide an indication of the risk associated with forcing a distributional assumption on the data. That is, if the parameter bounds are too wide and there is no general area of intersection, incorrect inferences may result (King 1997).

Despite the challenges that one faces when analyzing tomography plots, such inspection is worthwhile because it helps researchers to evaluate whether the ultimate conditional distributions are fairly close approximations to the truth. As such, we decided to create and examine tomography plots for every single dataset we considered and eventually selected. Our assessment of the tomography plots suggests that all datasets selected for analysis are fairly “informative” in the sense that all of the lines tend to intersect in one general area of the plot and the parameter bounds are fairly narrow (see Figure 1 for an illustration).<sup>5</sup>

[INSERT FIGURE 1 ABOUT HERE]

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<sup>5</sup>All the tomography plots can be made available in an online appendix.

## Results

Using the R packages *ei* (King and Roberts, 2012) and *eiPack* (Lau et al., 2006), we estimated vote choice for candidates across racial groups using precinct-level election data. For EI, we take the iterative approach that has been questioned by some. In this approach, we iteratively estimate how each racial group voted for each candidate. So in an election with three different racial groups and 7 different candidates we estimate a total of 21 EI models. In contrast, the RxC approach allows users to estimate all the models simultaneously. Recall, our overarching question is: Does the iterative approach over-estimate racially polarized voting (RPV) compared to the RxC approach?

Despite differences with model efficiency, and theoretical claims of aggregation bias when using the EI iterative approach, we find no statistically different vote estimates across the 14 elections and 78 candidates we examine in the EI versus RxC approach (all the results race by race and candidate by candidate can be seen in the Appendix tables 16 to 28. Simply stated, our analysis reveals that both methods lead to the exact same conclusions about vote choice and racially polarized voting.

Where differences do exist, there is no consistent pattern in whether EI or RxC produce higher or lower levels of racially polarized voting, contrary to the expectations by some scholars. In some instances EI might yield 1 points higher minority vote cohesion, but in other instances RxC estimates 2 points higher minority vote cohesion, and in every instance the minority vote estimates are statistically indistinguishable from one another. In



full, we estimated 193 racial group-candidate vote outcomes and found that in 73 percent of the cases, the difference between EI and RxC is within 2 points. More specifically, in 105 instances the difference in the vote choice estimate is less than 1 point, and in 35 instances the difference is only between 1 and 2 points. This suggests remarkable consistency across the two approaches. For the remaining 27 percent of the cases, only 11 of them—or 6 percent—produce estimates that are over 5 points different from one another, as summarized in Table 2.

[INSERT TABLE 2 ABOUT HERE]

Furthermore, we find no evidence that EI is more likely to produce results in favor of racially polarized voting. For example, in the first election we considered, EI reports slightly higher minority cohesion—84.04 (EI) to 82.94 (RxC)—for the Latino-preferred candidate. However, in the second election we examined RxC reports slightly higher minority cohesion—94.39 (EI) to 96.56 (RxC)—for the Latino preferred candidate. In 20 instances in which minority voters had a minority preferred candidate, EI produces higher minority cohesion 8 times and RxC produces higher minority cohesion 12 times (see table 3). In some instances this difference in “higher cohesion” amounts to less than a half-point difference such as the Latino candidate Torrico winning an estimated 18.24 percent of the Latino vote in RxC versus an estimated 17.85 percent under EI. Thus even where differences exist they are often negligible and would round to the same whole number. Likewise, we find no evidence that White bloc voting against minority-preferred candidates is stronger under EI as compared to RxC, with each method sometimes

producing slightly higher White bloc voting exactly half of the time (see Table 3).

[INSERT TABLE 3 ABOUT HERE]

Recall that our second research questions was: Are there systematic outcome differences between EI and RxC when analyzing elections with few candidates versus elections with multiple candidates? We might expect greater differences to emerge when there are more candidates than fewer candidates—the claim is that RxC is designed for this scenario whereas EI is more equipped in dealing with 2x2 datasets. Another way of stating this is: Do EI and RxC essentially produce the same results when there are two, or maybe three candidates, but start to diverge when six, seven or more than ten candidates are on the ballot?

In the first section of our analysis we compared EI and RxC with only two racial groups—Latinos and Whites—across eight elections in which the number of candidates on the ballot varied from two to twelve. The elections consist of contests in Los Angeles, CA; Orange County, CA; Corona, CA; Orange County, FL; Oceanside, CA; Vista, CA; and San Mateo, CA. This diversity allows us to assess whether the number of candidates impact the stability of EI and RxC estimates. Table 4 shows the co-ethnic minority preferred candidate for each one of the eight elections. Figure 2 visualizes the differences between method estimates by race for each election. As is illustrated, there is no detectable pattern that would lead one to conclude that the iterative EI is more likely to produce results in favor of racially polarized voting.

[INSERT TABLE 4 ABOUT HERE]

[INSERT FIGURE 2 ABOUT HERE]

So far we have only examined races with two racial groups (Latinos and Whites). In the next section we compare EI and RxC in six elections with more than two racial groups; two elections with Latinos, Asians, and Whites; three with Latinos, Blacks, and Whites; and one election with the four racial groups. This allows us to assess our third major question: Are there systematic outcome differences between EI and RxC when analyzing elections with more than two racial groups?

In addition to the examining elections with different racial group combinations, our data enables us to consider elections with as low as two and as high as twelve candidates so that we can continue to assess whether systematic differences exist between EI and RxC in much more complex environments. Tables 5, 6, 7 report the co-ethnic minority preferred candidate for each one of elections examined. Similarly, Figures 3 and 4 visualize the differences. Finally, Figure 5 presents a compiled visualization of all the races with more than two ethnic groups. The results show remarkable similarity between EI and RxC estimates even as the number of ethnic groups and candidates increases. Once again, we also find no pattern that would lead one to conclude that EI is more or less likely than RxC to produce results in favor of racially polarized voting patterns.

[INSERT TABLE 5 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

[INSERT FIGURE 4 ABOUT HERE]

[INSERT TABLE 6 ABOUT HERE]

[INSERT FIGURE 5 ABOUT HERE]

[INSERT TABLE 7 ABOUT HERE]

## **Comparing Iterative EI and RxC Estimates with Known Exit Poll Results**

In many if not most situations where analysts are called to evaluate the presence or absence of racially polarized voting, EI is the chosen method in part because individual-level polling data are unavailable. For instance, pollsters do not collect data for elections in small cities, such as Blythe, CA. In major cities, though, occasionally data are available.

While our main question is whether EI and RxC produce different RPV outcomes, there is a possibility that EI may be inaccurate relative to the “truth” more often than the RxC approach. To consider this possibility, we decided to compare EI and RxC estimates in a voting scenarios with known outcomes that provide vote choice by race (i.e., an exit poll or pre-election poll).

Many studies have pointed out that ecological fallacy and aggregation bias can produce ecological inference results that are questionable. While we acknowledge the limitations of ecological inference and problems of aggregation bias, we find that the results from EI and RxC are similar to

the individual-level exit poll data as it pertains to evaluating racially polarized voting patterns in voting rights act cases. Table 8 shows EI and RxC results for the 2005 Los Angeles mayoral runoff election between Antonio Villaraigosa (Latino) and James Hahn (white). These numbers are compared against results for the Los Angeles Times exit poll. The results demonstrate that not only do EI and RxC produce remarkably similar estimates, but that the estimates closely match the individual-level results from the Los Angeles Times poll. More specifically, the EI method estimates Villaraigosa receiving 82 percent of the Latino vote and only 45 percent of the white vote; the RxC method estimates Villaraigosa receiving 81 percent of the Latino vote and just 48 percent of the white vote. If the task is to evaluate a pattern of racially polarized voting, both methods closely match the conclusion one would draw from the exit poll, which reports that an estimated 84 percent of Latinos voted for Villaraigosa while only 50 percent of white voters made the same choice. While the EI method shows slightly more RPV compared to the RxC method, the difference is very minimal. Moreover, the EI and RxC estimates are all within the margin of error of the individual level data reported by the LA Times exit poll. In sum, this comparison provides additional evidence that both methods may be useful in evaluating RPV in voting rights act lawsuits.

[INSERT TABLE 8 ABOUT HERE]

## Model Congruence Score: Do the Two Methods Lead to Similar Conclusions?

The previous sections revealed that the EI and RxC methods tend to produce similar vote choice estimates – given that the truncated bivariate normal (TBVN) distribution assumption is reasonable for the dataset under consideration. However, another way to compare the two methods is to evaluate how congruent the set of results are across the two different ecological models. This is important, because plaintiffs must show judges that racially polarized voting exists, and that it is not just a function of choosing one statistical technique over another, but something that holds regardless of technique. There is no doubt that as scholars we are similarly interested in understanding the extent to which findings are the result of a particular model estimation, or more broadly consistent across different estimators.

In this section, we discuss a new approach to determine whether the two methods produce similar judgements in what we call the model congruence score (MCS). The MCS can be applied in either 2x2 settings or with some adjustments extended to situations with multiple candidates and multiple groups, although analysts may want to set some decision rules in terms of whether to combine all white or Latino candidates (for instance) into one racial group candidate (for the purpose of assessing racially polarized voting this is often instructed by the court).

What exactly do we want the MCS to tell us? First, do both approaches conclude that minority voters prefer the minority candidate and that white voters prefer the minority candidate? If minority voters prefer the white

candidate and so do white voters, then racially polarized voting does not exist. Likewise, if both minority and white voters both prefer the minority candidate, racially polarized voting does not exist. To answer this initial question, MCS rates whether or not simple polarized voting exists based on the estimates obtained from iterative EI and RxC.

Second, what is the relative degree of racially polarized voting in each of the models? That is, for example, do both models suggest a 30-point gap in racial voting preference, or does one model suggest only a 5-point difference and the second model suggests a 40-point difference? The difference in voting preferences and not just the direction of preferences is a very important component of the congruence score and informative to the courts. In order to answer this second question, MCS first estimates the percentage point-gap between minority and white voters for the minority preferred candidate, and then for the white-preferred candidate. Next, MCS evaluates what percentage of minority voters would need to switch from voting for the minority candidate to supporting the white candidate such that there is an even 50-50 distribution, and no set preferred candidate. Likewise, MCS calculates the percentage of white voters that would need to switch from voting for the white candidate to supporting the minority candidate to create a 50-50 distribution.

Third, if voting patterns hold, are minority voters blocked by white voters from electing a minority candidate? And by how much are they blocked? Again this step adds both a simple 'yes/no' distinction of being blocked, but also calculates and compares the degree by which a minority-preferred candidate is blocked. Overall the MCS attempts to provide a simple measure,

ranging from 0 to 1, to assess how much overall congruence there is between and within the vote choice estimates across EI and RxC.

We first calculated MCS for both EI and RxC in a simple 2x2 configuration to show in more detail how the process works. We report congruence scores for each metric, which is scaled from 0-1 where 0 reveals the two methods are in complete disagreement and 1 indicates the two methods are in complete agreement. For ease of interpretation, we explain the precise metrics for the aforementioned three tests and their congruence with actual data from EI and RxC estimates of the Latino and non-Latino vote from the 2010 Los Angeles County Insurance Commissioner race where the Latino candidate, De la Torre, ran against Jones (white). While the non-Latino group includes non-Latino minorities, for simplicity we bin whites with non-Latino minorities in order to craft a simple 2x2 (see Table 15 in the appendix for full vote choice estimates).

To assess whether Latino voters prefer the Latino candidate, we examine the difference between Latino support for De la Torre and white/other support for De la Torre. EI places Latino support for De la Torre at 84 percent, whereas for white/non-Latinos the estimate is at just 22 percent. The difference is thus just over 62 percent. The similar calculation is made for the RxC method, placing Latino support for De la Torre at about 83 percent, whereas non-Latino support falls at nearly 23 percent—a difference of 60 percentage points. How similar are these findings? To calculate the congruence score on this measure we take the absolute difference between the EI and RxC estimate for minority-white support for De la Torre then divide this by the absolute mean difference of the two methods. Finally, to put this



on a 0-1 scale where 1 equals pure congruence and 0 equals no congruence, we subtract the result from value 1 so that values closer to 1 imply higher congruence:

$$\begin{aligned}
 x &= \text{EI Latino vote for De la Torre} - \text{EI Non-Latino vote for De la Torre} \\
 y &= \text{RxC Latino vote for De la Torre} - \text{RxC Non-Latino vote for De la Torre} \\
 &= 1 - \frac{\text{abs}(x - y)}{\text{abs}(\text{mean}(x, y))}
 \end{aligned} \tag{1}$$

We can plug the data from Table 15 into the above equation to produce the congruence score, which is identical to the congruence score appearing on row one of Table 9:

$$\begin{aligned}
 &= 1 - \frac{\text{abs}((84.11 - 22.02) - (82.94 - 22.99))}{\text{abs}(\text{mean}((84.11 - 22.02), (82.94 - 22.99)))} \\
 &= 0.965
 \end{aligned} \tag{2}$$

Row two in Table 9 assesses whether De la Torre is preferred by Latino voters. The congruence receives 1 if both the EI and RxC method reveal that Latinos preferred De la Torre to Jones (or 1 if both methods revealed a preference for Jones). In the present case, both methods show that Latinos prefer De la Torre, so the congruence on this metric receives a 1. The preference rate is calculated as the difference between Latino support for the Latino candidate, De la Torre, and the white candidate, Jones. For EI, this would be 84.11 - 15.92. The resulting figure is then divided by 2, to show

how much above the 50 percent mark De la Torre is preferred over Jones. In other words, what is the percentage of Latino voters who would have to switch to Jones so that Latinos did not prefer either candidate? For EI, this number is 34. Using the same calculation for RxC, we arrive at nearly 33. Thus, our numbers are very similar, and so a congruence score of 0.966 is reported. The equations for this congruence are listed below:

$$\begin{aligned}
 x &= (\text{EI Latino vote for De la Torre} - \text{EI Latino vote for Jones})/2 \\
 y &= (\text{RxC Latino vote for De la Torre} - \text{RxC Non-Latino vote for Jones})/2 \\
 &= 1 - \frac{\text{abs}(x - y)}{\text{abs}(\text{mean}(x, y))}
 \end{aligned} \tag{3}$$

The actual numbers are presented here:

$$\begin{aligned}
 x &= (84.11 - 15.92)/2 \\
 y &= (82.94 - 17.06)/2 \\
 &= 1 - \frac{\text{abs}(x - y)}{\text{abs}(\text{mean}(x, y))} \\
 &= 0.966
 \end{aligned} \tag{4}$$

Finally, we turn to vote blocking. Given the way districts are often drawn, this is a crucial question posed to judges who assess whether whites are blocking Latinos from electing their preferred candidates (usually Latino). In our working example, for non-Latinos we subtract their support for Jones

from non-Latino support for De la Torre. This is then divided by two (as in the above set of equations). This essentially measures how much whites (or non-Latinos) support the white candidate, and how much vote they would need to dole out to the Latino candidate to not be blocking the Latino candidate from getting elected. For EI, this is  $(22 - 78)/2$ , and for RxC this is  $(23 - 77)/2$ . Once again, the congruence score is calculated in a similar way as above, with a very high score of 0.965. Row four of Table 9 also reports whether whites are, in general, block voting against Latinos – if both the EI and RxC agree, then the congruence is given a 1.

$$\begin{aligned}
 x &= (\text{EI Non-Latino vote for De la Torre} - \text{EI Non-Latino vote for Jones})/2 \\
 y &= (\text{RxC Non-Latino vote for De la Torre} - \text{RxC Non-Latino vote for Jones})/2 \\
 &= 1 - \frac{\text{abs}(x - y)}{\text{abs}(\text{mean}(x, y))}
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 x &= (22.02 - 77.99)/2 \\
 y &= (22.98 - 77.01)/2 \\
 &= 1 - \frac{\text{abs}(x - y)}{\text{abs}(\text{mean}(x, y))} \\
 &= 0.965
 \end{aligned} \tag{6}$$

For the total minority candidate congruence score, we take the mean of the existing congruence scores, resulting in a final score of 0.9792. The process is reversed for calculating the requisite scores for the Anglo candidate.

In the 2x2 scenario, the numbers are essentially the same as those calculated for the minority candidate; however the coefficient sign is switched, and the block rate and preference rates are swapped. The final step taken to obtain an “overall” or “total model congruence score” is to then calculate the average of the minority candidate and white candidate congruence scores obtained in the previous steps (see result below).

[INSERT TABLE 9 ABOUT HERE]

Beyond the 2x2 example presented above, we also provide detailed model congruence scores for a 2x4, a 2x5, a 3x2 and a 4x7 election analysis comparing EI and RxC, and then a summary table which reports the final model congruence score for all 14 elections in our study. The full vote choice results of these elections can be found in Tables 15 – 28 but in this section we detail how congruent the ecological estimates are across the EI and RxC models. We should note that as more candidates are added the process becomes more complex, but the same underlying principles highlighted in the 2x2 case apply.

Tables 9 – 13 show that across different types of elections – some with more candidates and some with more groups of voters – the iterative EI and simultaneous RxC are very congruent. The model congruence scores capture the relative rank of the candidates, the size of the gap between first and second choice, and the size of the gap between minorities and whites. In the examples provided below, we find very strong evidence of model congruence across different election settings. Finally, the overall congruence scores for all elections analyzed are reported in Table 14, once again showing overall

high levels of congruence.

[INSERT TABLE 10 ABOUT HERE]

[INSERT TABLE 11 ABOUT HERE]

[INSERT TABLE 12 ABOUT HERE]

[INSERT TABLE 13 ABOUT HERE]

[INSERT TABLE 14 ABOUT HERE]

## Conclusion

This paper engages an important methodological question as to whether substantive differences emerge across two common methods used to estimate individual-level behavior from aggregate-level data. Specifically, we examined three questions: 1) Does EI's iterative technique over-estimate racially polarized voting (RPV) compared to RxC? In other words, does EI bias towards detecting RPV? 2) Are there systematic outcome differences between EI and RxC when analyzing elections with few candidates versus elections with multiple candidates? 3) Are there systematic outcome differences between EI and RxC when analyzing elections with more than two racial groups?

To assess whether voting districts experience racially polarized voting, we estimate vote share for different candidates from voters of different racial groups using two ecological inference methods. We evaluated King's iterative ecological inference (EI) approach against the more recent rows by

columns (RxC) approach. Using elections with multiple candidates and multiple groups (i.e., Latinos, whites, blacks, Asians), we find that no significant differences emerge across the two methods. Furthermore, to the extent differences do emerge, they are not systematic. Our general conclusions of whether racially polarized voting exists in a particular voting jurisdiction is the same for the two methods. Further, we presented a new congruence test that analysts can use to assess racially polarized voting when using both EI and RxC methods.

These are important findings to scholars of voting behavior, especially academics and practitioners who evaluate litigation in the voting rights arena. While there has been a robust debate on precisely what method to use, our results suggest both methods are similar, given that model assumptions are met. Moreover, our approach allows scholars to easily compare the results of the two methods, which, in the end also serves as a robustness check.<sup>6</sup>

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<sup>6</sup>We will post our R package to CRAN so that other researchers can employ similar analyses.

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# 1 Tables

Table 1: Summary Table of Elections Analyzed

Geography	Year	Ethnic Grps	# Cand.	Contest	Precincts
Los Angeles Co., CA	2010	2 (L, W)	2	Insurance Commissioner Dem Primary	4,980
Orange Co., FL	2006	2 (L, W)	3	School Board	44
Corona, CA	2006	2 (L, W)	4	City Council	47
Orange Co., FL	2012	2 (L, W)	5	County Commission	38
Corona, CA	2004	2 (L, W)	6	City Council	48
Oceanside, CA	2012	2 (L, W)	7	City Council	78
Vista, CA	2012	2 (L, W)	9	City Council	36
San Mateo, CA	2010	2 (L, W)	12	Superintendent of Public Education	433
Orange Co., CA	2010	3 (L, W, A)	2	Insurance Commissioner Dem Primary	1,941
Fullerton, CA	2012	3 (L, W, A)	12	City Council	84
Harris Co., TX	2010	3 (L, W, B)	2	Land Commissioner	885
Harris Co., TX	2010	3 (L, W, B)	3	Lieutenant Governor Dem Primary	885
Orange Co., FL	2008	3 (L, W, B)	4	Soil & Water Board of Directors	252
Los Angeles Co., CA	2010	A (L, W, B, A)	7	Attorney General Dem Primary	4,974

*Note: L= Latino, W=White, B=Black, A=Asian*

Table 2: Distribution of difference between EI and RxC vote choice estimates

EI vs. RxC outcome	n	%
Less than 1 point difference	105	54%
1 to 2 points difference	35	18%
2 to 3 points difference	19	10%
3 to 4 points difference	8	4%
4 to 5 points difference	15	8%
Over 5 points difference	11	6%
<i>Out of 193 vote choice scenarios</i>		

Table 3: Comparison of which method produces stronger racially polarized voting estimates in conditions with minority-preferred candidate

	Minority cohesion	White bloc voting
EI stronger polarization	8	10
RxC stronger polarizaton	12	10
<i>Out of 20 instances where minority voters had a minority preferred candidate</i>		

Table 4: Elections with 2 Ethnic Groups (Latino &amp; White)

Geography	# of Candidates	EI vs RxC estimate difference	
		Latinos	Whites
Los Angeles Co., CA	2	-1.10	1.04
Orange Co., FL	3	2.17	-0.75
Corona, CA	4	-0.96	0.29
Orange Co., FL	5	2.78	-0.73
Corona, CA	6	0.76	-0.11
Oceanside, CA	7	-4.52	0.93
Vista, CA	9	1.05	-0.31
San Mateo, CA	12	-1.32	0.16

Table 5: Elections with 3 Ethnic Groups (Latino Blacks, &amp; White)

Geography	# of Candidates	EI vs RxC estimate difference		
		Latinos	Whites	Blacks
Harris CO, TX	2	-4.62	-8.59	4.63
Orange Co., FL	4	0.14	-1.20	-3.76
Harris CO, TX	3	0.01	1.73	-4.65

Table 6: Elections with 3 Ethnic Groups (Latino, Asian &amp; White)

Geography	# of Candidates	EI vs RxC estimate difference		
		Latinos	Whites	Asians
Orange Co., CA	2	2.95	-0.90	-6.78
Fullerton, CA	12	1.72	-0.80	2.79

Table 7: Elections with 4 Ethnic Groups (Latino, Black, Asian, &amp; White)

Geography	# of Candidates	EI vs RxC estimate difference			
		Latinos	Whites	Asians	Blacks
Los Angeles Co., CA	7	1.23	1.19	2.54	-2.85



Table 8: Percent voting for Antonio Villaraigosa (AV) and James Hahn (JH) by ethnic group. Comparison between EI, RxC, and exit poll methods, Los Angeles mayoral election runoff, May 2005. Exit poll taken from Los Angeles Times.

	EI: AV	EI: JH	RxC: AV	RxC: JH	Exit: AV	Exit: JH	MOE
White	45	54	48	52	50	50	+/- 2.5
Black	58	40	50	50	48	52	+/-4.2
Latino	82	17	81	19	84	16	+/-3.6
Asian	48	51	47	53	44	56	+/-6.1

Table 9: 2x2 Congruence table for Los Angeles County Insurance Commissioner Election 2010

	EI	RxC	Congruence
MV1-WV for MC1	62.091	59.955	0.965
MC1 preferred by MV1	Yes	Yes	1
MC1 preference rate	34.094	32.946	0.966
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-27.986	-27.012	0.965
MC Model Congruence			0.9792
MV1-WV for WC1	-62.091	-59.955	0.965
WC1 preferred by WV1	Yes	Yes	1
WC1 preference rate	27.986	27.012	0.965
WC1 blocked by MV1	Yes	Yes	1
WC1 block rate	-34.094	-32.946	0.966
WC Model Congruence			0.9792
Total Model Congruence Score			0.9792

Note: see Table 15 for actual polarized voting results for EI and RxC

Table 10: 2x4 Congruence table for Corona, CA City Council 2006

	EI	RxC	Congruence
MV1-WV for MC1	15.8	14.5	0.9142
MC1 preferred by MV1	Yes	Yes	1
MC1 pref rate	3.8	3.4	0.8873
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-5.4	-5.3	0.9813
MC model congruence score			0.9566
MV1-WV for WC1	-2.5	-2.8	0.8868
WC1 preferred by WV	Yes	Yes	1
WC1 pref rate	2.05	2.1	0.9759
WC1 blocked by MV1	Yes	Yes	0.5
WC1 block rate	-3.75	-3.4	0.8873
WC model congruence score			0.85
Total model congruence score			0.9033

Note: see Table 17 for actual polarized voting results for EI and RxC

Table 11: 2x5 Congruence table for Orange County, FL Commissioner 2012

	EI	RxC	Congruence
MV1-WV for MC1	32.5	36	0.8978
MC1 preferred by MV1	Yes	Yes	1
MC1 pref rate	5.3	7.2	0.6867
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-17.3	-16.7	0.9646
MC model congruence score			0.9098
MV1-WV for WC1	-26.5	-19.8	0.7106
WC1 preferred by WV	Yes	Yes	1
WC1 pref rate	7	6.1	0.8544
WC1 blocked by MV1	Yes	Yes	0.8
WC1 block rate	-12.25	-11.3	0.9149
WC model congruence score			0.856
Total model congruence score			0.8829

Note: see Table 18 for actual polarized voting results for EI and RxC

Table 12: 3x2 Congruence table for Harris County, TX Lt. Gov Dem Primary 2010

	EI	RxC	Congruence
MV1 - Latinos			
MV1-WV for MC1	60.3	64.3	0.9358
MC1 preferred by MV1	Yes	Yes	1
MC1 pref rate	23.8	19.2	0.7855
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-36.6	-45.2	0.7908
MC1 model congruence score			0.9024
WV - Whites			
MV1-WV for WC1	-60.4	-64.3	0.9374
WC1 preferred by WV	Yes	Yes	1
WC1 pref rate	36.6	45.2	0.7908
WC1 blocked by MV1	Yes	Yes	1
WC1 block rate by MV1	-23.8	-19.2	0.7855
WC model congruence score			0.9028
MV2 - Blacks			
MV2-WV for MC1	81.6	94.8	0.8503
MC1 preferred by MV2	Yes	Yes	1
MC1 pref rate	45.1	49.7	0.9029
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-36.6	-45.2	0.7908
MC1 model congruence score			0.9088
WV - Whites			
MV2-WV for WC1	-81.7	-94.8	0.8516
WC1 blocked by MV2	Yes	Yes	1
WC1 block rate by MV2	-45.1	-49.7	0.9029
WC model congruence score			0.9182
Total model congruence score			0.9081

Note: see Table 26 for actual polarized voting results for EI and RxC

Table 13: 4x7 Congruence Table for Los Angeles County, CA Primary election for Attorney General 2010

	EI	RxC	Congruence
MV1 - Latinos			
MV1-WV for MC1	28.3	28.3	1
MC1 preferred by MV1	Yes	Yes	1
MC1 pref rate	10.7	11.1	0.9633
MC1 blocked by WV	Yes	Yes	1
MC1 block rate	-11.5	-10.9	0.9464
MC1 model congruence score			0.9819
WV - Whites			
MV1-WV for WC1	-25.1	-29.7	0.8321
WC1 preferred by WV	Yes	Yes	1
WC1 pref rate	8.1	8.6	0.9459
WC1 blocked by MV1	Yes	Yes	1
WC1 block rate by MV1	-15.2	-18.1	0.8258
WC model congruence score			0.9208
MV2 - Blacks			
MV2-WV for MC2	29.1	38.8	0.7143
MC2 preferred by MV2	Yes	Yes	1
MC2 pref rate	25.4	31.7	0.7789
MC2 blocked by WV	No	No	1
MC2 block rate*	8.1	8.6	0.9459
MC2 model congruence score			0.8878
WV - Whites			
MV2-WV for WC1	-29.1	-38.8	0.7143
WC1 blocked by MV2	No	No	1
WC1 block rate by MV2	25.4	31.7	0.7789
WC model congruence score			0.8311
MV3 - Asians			
MV3-WV for MC3	11.2	11.6	0.9649
MC3 preferred by MV3	Yes	Yes	1
MC3 pref rate	5.7	3.2	0.4237
MC3 blocked by WV	Yes	Yes	1
MC3 block rate	-8.2	-9.4	0.8689
MC3 model congruence score			0.8515
WV - Whites			
MV3-WV for WC1	-16.6	-13.4	0.7867
WC1 blocked by MV3	Yes	Yes	1
WC1 block rate by MV3	-5.7	-3.2	0.4237
WC model congruence score			0.7368
Total model congruence score			0.8717

Note: see Table 28 for actual polarized voting results for EI and RxC

Table 14: Summary of overall model congruence scores across all elections analyzed

RxC	Georgraphy	Precinct (n)	Congruence
2x2	Los Angeles, CA	4980	0.9792
2x3	Orange County, FL	44	0.9818
2x4	Corona, CA	47	0.9033
2x5	Orange County, FL	38	0.8829
2x6	Corona, CA	48	0.8546
2x7	Oceanside, CA	78	0.7857
2x9	Vista, CA	36	0.9377
2x12	San Mateo, CA	433	0.9561
3x2	Orange County, CA	1941	0.8169
3x12	Fullerton, CA	84	0.8344
3x2	Harris County, TX	885	0.9081
3x3	Harris County, TX	885	0.7952
3x4	Orange County, FL	252	0.8695
4x7	Los Angeles, CA	4974	0.8717

## 2 Figures

Figure 1: Sample Tomography Plots

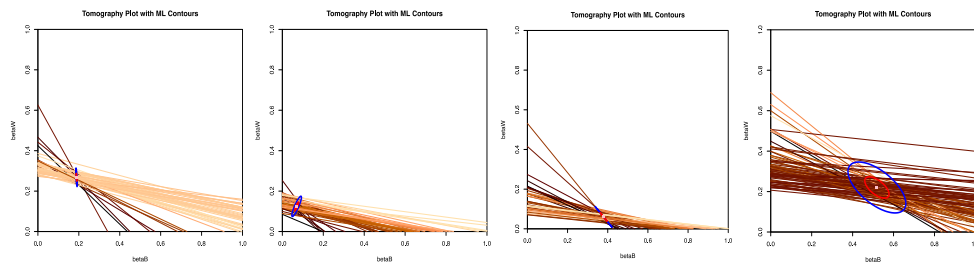




Figure 2

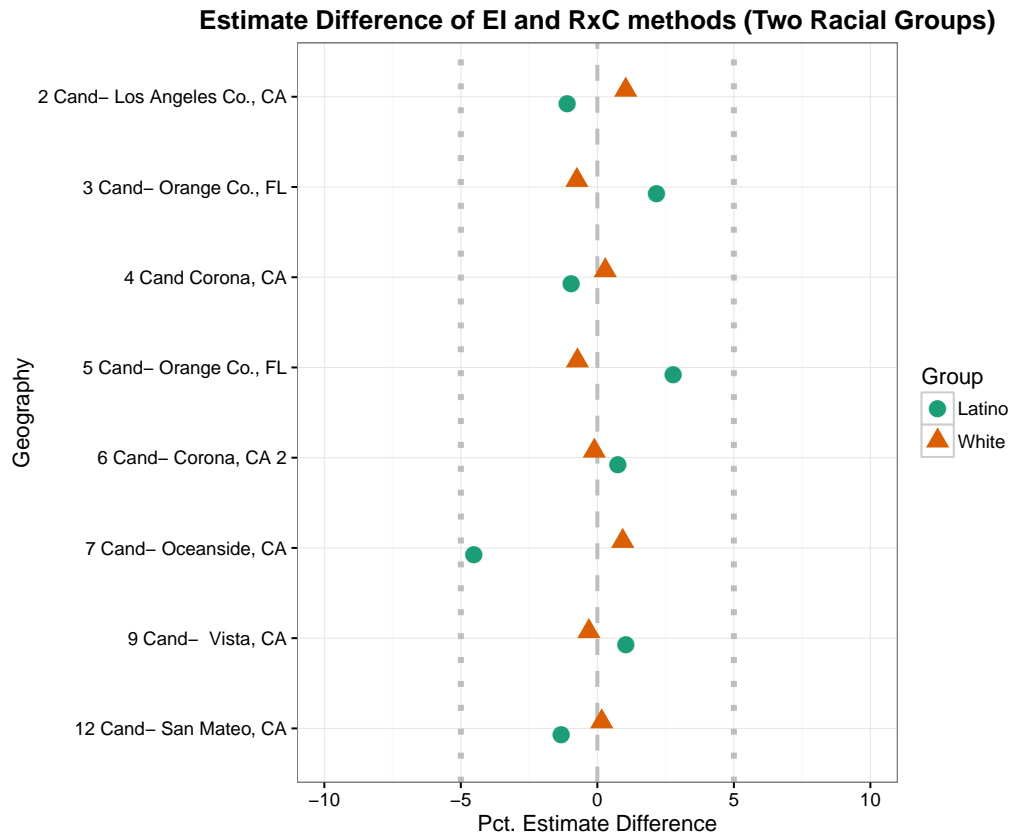


Figure 3

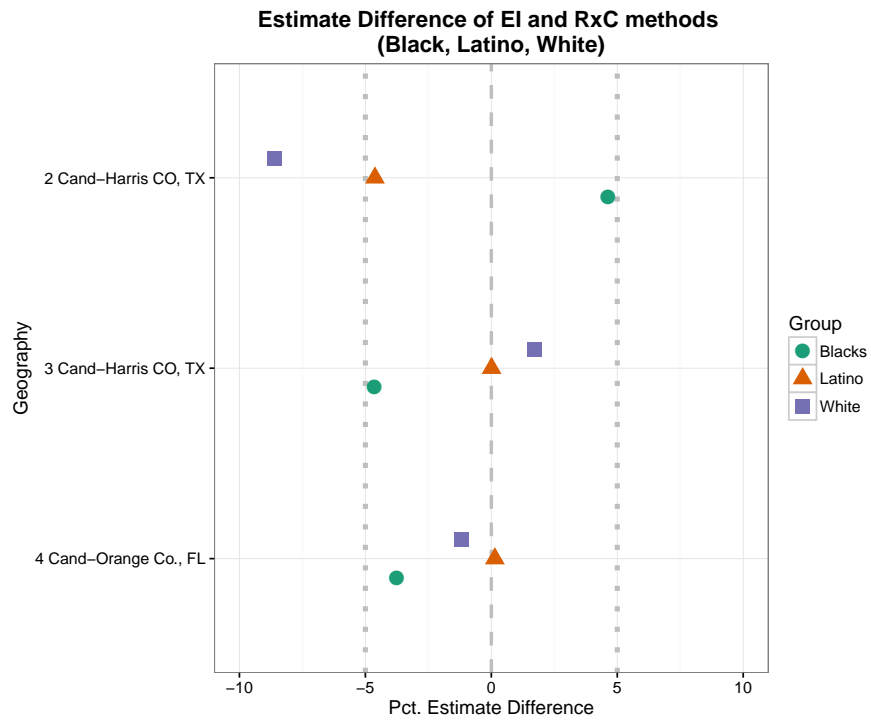


Figure 4

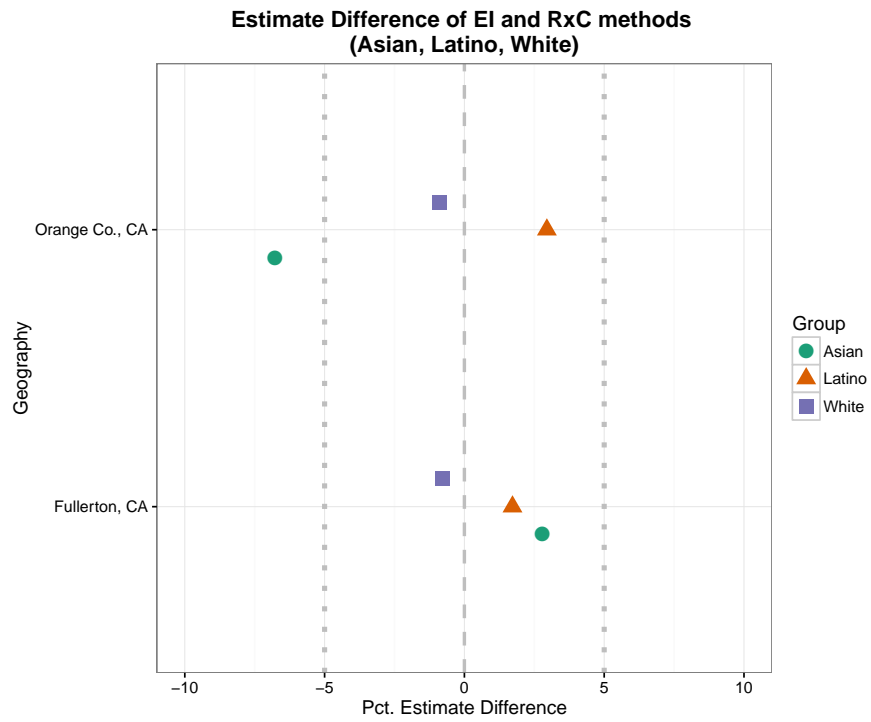
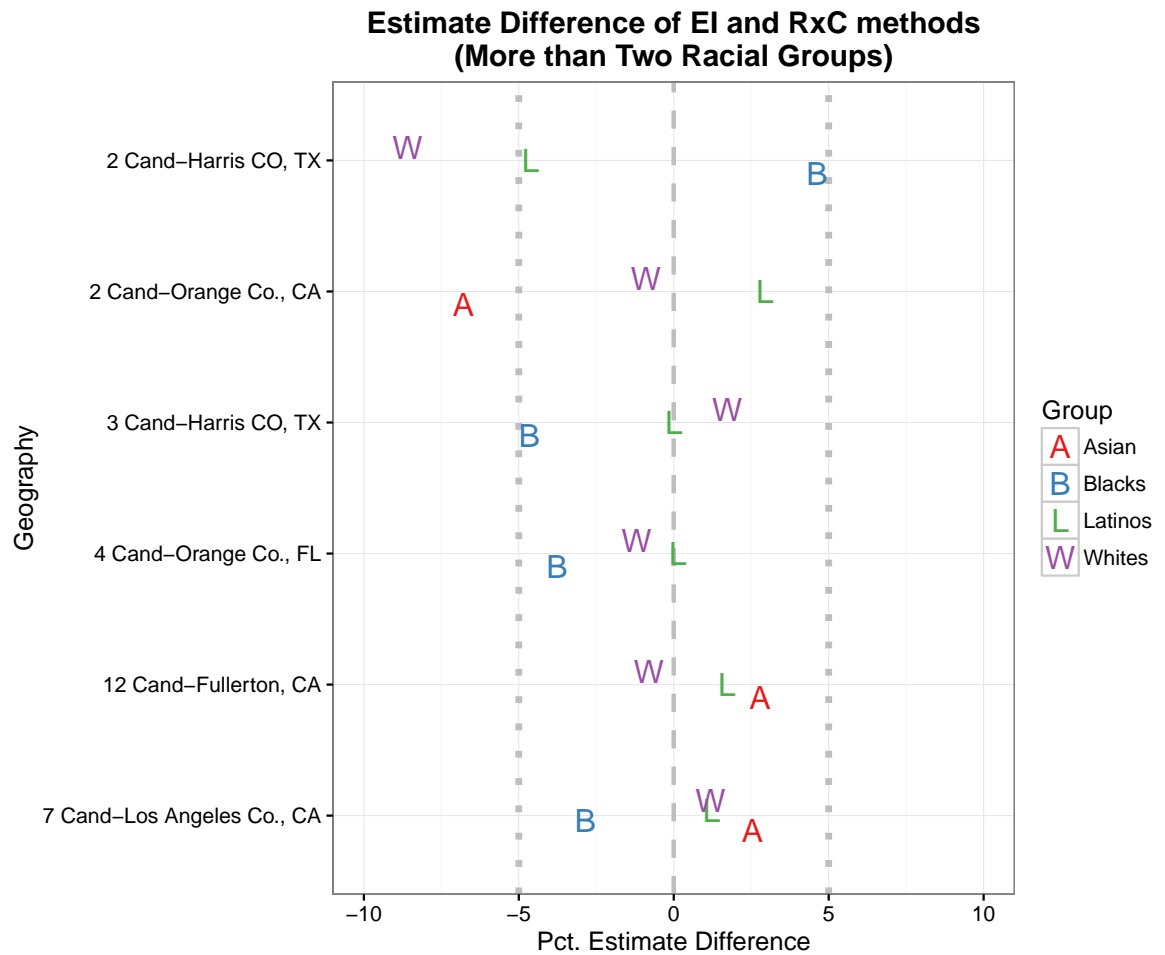


Figure 5



## A Latino vs. Non-Latino

Table 15: Los Angeles County, CA Insurance Commissioner 2010 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% De la Torre	84.11	82.94	-1.17	22.02	22.99	0.97
se	9.49	0.59		7.11	0.45	
% Jones	15.92	17.05	1.13	77.99	77.01	-0.98
se	9.51	0.58		7.11	0.45	
Total	100.03	100.00	-0.04	100.01	100.00	-0.01
Precinct n = 4980, Number of Candidates = 2						

Table 16: Orange County, Florida School Board 2006 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Flynn	0.8	3.6	2.8	57.8	57.7	-0.1
se	0.9	3.4		0.0	1.5	
% Kelly	15.7	18.7	3.0	32.4	30.5	-1.9
se	2.5	7.2		0.7	1.9	
% Cardona	94.3	96.5	2.2	8.1	7.4	-0.7
se	4.2	2.7		1.0	0.9	
Total	110.9	118.9	8.0	98.4	95.7	-2.7
Precinct n = 44, Number of Candidates = 3						

Table 17: Corona, CA City Council 2006 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Breitenbucher	19.6	18.1	-1.5	21.1	21.5	0.4
se	0.7	1.6		0.1	0.5	
% Montanez	35.9	34.9	-0.1	20.1	20.4	0.3
se	0.02	1.70		0.05	0.56	
% Spiegel	28.4	28.2	-0.2	30.9	31.0	0.1
se	0.6	1.1		0.3	0.3	
% Skipworth	18.8	18.6	-0.2	26.8	26.9	0.1
se	0.8	1.7		0.4	0.5	
Total	102.9	100.0	-2.9	99.1	100.0	0.9
Precinct n = 47, Number of Candidates = 4						

Table 18: Orange County, Florida 2012 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Clarke	24.7	23.6	-1.1	23.2	23.2	0.0
se	10.3	3.3		3.7	1.4	
% Damiani	10.7	15.5	4.8	37.2	35.3	-1.9
se	2.8	6.1		0.9	2.6	
% Lasso	13.3	12.2	-1.1	15.4	16.1	0.7
se	2.3	2.0		2.3	0.8	
% Aviles	35.2	38.0	2.8	2.7	2.0	-0.7
se	5.0	2.1		1.5	0.8	
% Pisano	12.0	11.0	-1.0	22.5	23.1	0.6
se	0.8	5.4		0.1	2.4	
Total	96.1	100.6	4.5	101.2	99.8	-1.4

Precinct n = 38, Number of Candidates = 5

Table 19: Corona, CA City Council 2004 EI vs. EI:RxC Comparison

Candidate	Latino Vote		Diff	Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>		<b>EI</b>	<b>RxC</b>	Diff
% Miller	20.7	15.9	-4.8	28.2	29.3	1.1
se	10.1	4.4		2.5	1.3	
% Melendez	38.5	39.2	0.7	4.5	4.4	-0.1
se	2.0	1.9		0.8	0.6	
% Nolan	18.6	16.3	-2.3	25.7	26.4	0.7
se	0.1	3.4		0.1	1.0	
% Humphrey	7.1	6.8	-0.3	12.4	12.5	0.1
se	1.7	2.3		0.4	0.6	
% Schnbal	2.5	3.0	0.5	8.5	8.6	0.1
se	2.1	1.1		0.7	0.3	
% Bennett	18.5	18.5	0.0	18.5	18.5	0.0
se	2.7	2.6		0.7	0.7	
Total	106.0	99.9	-6.1	97.9	100.0	2.1
Precinct n = 48, Number of Candidates = 6						



Table 20: Oceanside, CA City Council 2012 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Dykes	0.8	2.0	1.2	17.8	17.6	-0.2
se	0.8	1.7		0.0	0.6	
% Corso	9.4	15.8	6.4	20.8	21.9	1.1
se	3.8	3.7		0.4	0.8	
% Zerinek	8.3	9.1	0.8	6.7	6.5	-0.2
se	0.9	1.3		0.1	0.3	
% Snyder	6.8	6.6	-0.2	1.4	1.7	0.3
se	0.7	0.6		0.7	0.1	
% Sanchez	53.1	48.5	-4.6	21.8	22.7	0.9
se	8.2	4.5		2.0	1.0	
% Feller	7.6	10.7	3.1	25.5	24.7	-0.8
se	3.8	4.1		1.0	0.9	
% Knott	12.0	12.5	0.5	3.7	3.6	-0.1
se	1.0	1.1		0.2	0.2	
Total	98.2	105.6	7.4	98.0	98.9	0.9
Precinct n = 78, Number of Candidates = 7						

Table 21: Vista, CA City Council 2012 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% YoungRigby	9.0	8.7	-0.3	17.0	17.1	0.1
se	0.9	2.0		0.3	0.5	
% Miles	9.9	8.9	-1.0	3.0	3.2	0.2
se	1.5	1.4		0.3	0.3	
% Kaiser	2.5	2.8	0.3	18.5	18.3	-0.2
se	1.6	1.8		0.4	0.5	
% Campbell	15.0	14.9	-0.1	18.6	18.6	0.0
se	4.2	1.8		1.0	0.5	
% Lopez	37.9	38.9	1.0	6.0	5.6	-0.4
se	0.1	1.7		0.1	0.4	
% Garretson	2.7	2.6	-0.1	11.9	11.4	-0.5
se	2.3	2.1		0.1	0.7	
% Ford	7.5	2.3	-5.2	5.0	7.3	2.3
se	0.4	1.8		0.3	0.6	
% Staight	8.3	8.2	-0.1	3.3	3.4	0.1
se	1.5	1.2		0.2	0.3	
% Fleming	23.1	19.4	-3.7	13.0	13.2	0.2
se	8.3	3.4		2.0	0.9	
Total	116.2	107.0	-9.2	96.7	98.4	1.7
Precinct n = 36, Number of Candidates = 9						

Table 22: San Mateo, CA 2010 Primary EI vs. EI:RxC Comparison

Candidate	Latino Vote			Non-Latino Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Gutierrez	32.8	27.7	-5.1	7.1	7.4	0.3
se	20.7	2.1		1.9	0.3	
% Lenning	5.3	1.6	-3.7	3.2	3.6	0.4
se	4.6	1.0		0.4	0.1	
% Martin	0.0	1.8	1.8	2.3	2.1	-0.2
se	0.0	0.7		0.0	0.1	
% McMicken	7.2	9.6	2.4	7.0	6.7	-0.3
se	4.7	1.4		0.4	0.2	
% Deligianni	2.6	2.8	0.2	4.9	4.9	0.0
se	2.2	1.2		0.2	0.1	
% Shiehk	0.9	0.4	-0.5	0.6	0.6	0.0
se	0.8	0.2		0.1	0.0	
% Nusbaum	1.2	4.1	2.9	3.2	3.4	0.2
se	1.1	1.0		0.5	0.1	
% Romero	43.1	41.8	-1.3	17.8	18.0	0.2
se	15.3	2.7		1.8	0.4	
% Blake	0.8	0.6	-0.2	5.7	5.9	0.2
se	0.8	0.5		0.1	0.2	
% Williams	0.1	2.5	2.4	1.6	1.4	-0.2
se	0.0	0.6		0.1	0.1	
% Torlakson	8.2	8.3	0.1	27.3	27.3	0.0
se	7.1	3.7		0.8	0.5	
% Aceves	1.4	5.3	3.9	17.9	17.5	-0.4
se	0.9	2.7		0.2	0.4	
Total	104.0	107.14	3.1	99.1	99.3	0.2
Precinct n = 433, Number of Candidates = 12						

## B Latino, Asian, & White

Table 23: Orange County, CA Insurance Commissioner 2010 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Asian Vote			White Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Jones	11.9	8.9	-3.0	54.6	61.6	7.0	64.9	65.8	0.9
se	10.1	1.8		12.1	1.6		4.5	0.3	
% Delatorre	88.0	90.9	2.9	45.1	38.3	-6.8	35.0	34.1	-0.9
se	10.1	1.8		12.1	1.6		4.5	0.3	
Total	100.0	99.9	-0.1	99.7	100.0	0.3	99.9	100.0	0.1

Precinct n = 1941, Number of Candidates = 2

Table 24: Fullerton City, CA City Council 2012 EI vs. EI:RxC Comparison

Candidate	Latino Vote			Asian Vote			White Vote		
	EI	RxC	Diff	EI	RxC	Diff	EI	RxC	Diff
% Jaramillo	21.5	13.7	-7.8	1.9	1.9	0.0	4.3	6.3	2.0
se	4.2	2.6		1.6	1.6		2.0	0.8	
% Hakim	12.0	7.6	-4.4	7.6	3.0	-4.6	2.8	3.6	0.8
se	3.3	2.4		2.1	2.1		0.8	0.8	
% Alvarez	17.5	19.2	1.7	6.1	8.9	2.8	8.2	7.4	-0.8
se	6.1	1.6		4.0	1.9		2.5	0.5	
% Reid	4.7	4.8	0.1	1.9	1.5	-0.4	1.1	0.8	-0.3
se	1.3	0.4		1.1	0.5		0.5	0.1	
% Kiger	8.2	9.9	1.7	17.8	16.8	-1.0	11.5	10.7	-0.8
se	5.2	1.7		6.4	2.1		2.2	0.6	
% Levinson	1.7	1.6	-0.1	14.4	10.2	-4.2	7.2	7.3	0.1
se	1.4	0.9		9.8	1.3		1.5	0.3	
% Bartholomew	3.8	5.5	1.7	5.6	6.6	1.0	6.0	5.3	-0.7
se	2.8	1.0		4.1	1.2		1.2	0.3	
% Whitaker	12.2	13.6	1.4	20.1	20.9	0.8	13.0	12.7	-0.3
se	6.4	1.5		0.1	1.9		2.3	0.5	
% Bankhead	5.6	4.7	-0.9	7.1	6.4	-0.7	7.2	7.2	0.0
se	4.2	1.1		1.3	1.4		1.4	0.4	
% Flory	11.5	7.9	-3.6	6.5	4.8	-1.7	12.6	13.5	0.9
se	3.0	2.3		0.7	2.5		2.0	0.8	
% Rands	2.9	3.2	0.3	13.2	11.7	-1.5	8.2	8.4	0.2
se	2.3	1.5		4.0	2.0		2.0	0.5	
% Fitzgerald	7.7	7.6	-0.1	13.4	12.4	-1.0	14.9	15.4	0.5
se	5.8	2.2		2.6	2.7		1.7	0.7	
Total	109.8	99.9	-9.9	116.0	105.6	-10.4	97.5	99.2	1.7

Precinct n = 84, Number of Candidates = 12

## C Latino, Black, & White

Table 25: Harris County, TX 2010 General EI vs. EI:RxC Comparison

Candidate	Latino Vote			Black Vote			White Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Uribe	73.7	69.1	-4.6	95.0	99.6	4.6	13.4	4.8	-8.6
se	11.2	1.0		4.4	0.3		8.1	1.1	
% Patterson	26.2	30.8	4.6	4.9	0.3	-4.6	86.6	95.1	8.5
se	11.2	0.9		4.4	0.3		8.1	1.1	
Total	100.0	100.0	0.0	99.9	100.00	0.1	100.0	99.9	-0.1

Precinct n = 885, Number of Candidates = 2

Table 26: Harris County, TX 2010 Primary EI vs. EI:RxC Comparison

Candidate	Latino Vote			Black Vote			White Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Earle	28.8	28.2	-0.6	45.7	50.5	4.8	53.0	53.4	0.4
se	13.1	1.1		7.4	1.3		11.0	1.2	
% Katz	7.5	12.7	5.2	7.6	12.0	4.5	17.3	15.0	-2.3
se	2.7	0.7		3.0	0.9		6.0	0.8	
% Chavez	59.0	59.0	0.0	42.0	37.4	-4.6	29.7	31.5	1.8
se	12.9	1.1		0.0	1.4		10.5	1.2	
Total	95.4	99.9	4.5	95.3	99.9	4.7	100.1	100.0	-0.1

Precinct n = 885, Number of Candidates = 3

Table 27: Orange County, FL 2008 Soil/Water Board EI vs. EI:RxC Comparison

Candidate	Latino Vote			Black Vote			White Vote		
	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff	<b>EI</b>	<b>RxC</b>	Diff
% Cardona	65.7	65.8	0.1	24.2	20.5	-3.7	17.7	16.5	-1.2
se	6.0	0.9		2.7	0.6		3.0	0.3	
% Hamada	19.3	20.1	0.8	32.6	34.5	1.9	29.9	31.0	1.1
se	1.4	1.1		2.2	0.8		0.5	0.4	
% Whiting	2.7	2.5	-0.2	14.2	14.8	0.6	26.0	27.0	1.0
se	2.1	0.9		1.5	0.7		2.2	0.3	
% Hamilton	11.0	11.4	0.4	29.0	30.0	1.0	24.5	25.4	0.9
se	2.6	0.9		0.6	0.6		2.6	0.3	
Total	98.8	100.0	1.2	100.2	99.9	-0.3	98.2	100.0	1.8

Precinct n = 252, Number of Candidates = 4

## D Latino, Black, Asian, & White

Table 28: Los Angeles, CA 2010 State Attorney (General) EI vs. EI:RxC Comparison

Candidate	Latino Vote			Black Vote			Asian Vote			White Vote		
	EI	RxC	Diff	EI	RxC	Diff	EI	RxC	Diff	EI	RxC	Diff
% Harris	8.8	4.2	-4.6	63.0	72.7	9.7	17.3	20.5	3.2	33.9	33.9	0.0
se	6.1	0.4		18.9	0.6		14.8	0.8		9.6	0.3	
% Delgadillo	39.2	40.4	1.2	12.3	9.4	-2.9	11.8	14.4	2.5	10.9	12.1	1.2
se	8.0	0.3		4.2	0.4		4.2	0.6		5.8	0.2	
% Lieu	4.3	3.7	-0.6	0.8	0.8	0.0	28.7	26.8	-1.9	17.5	15.2	-2.3
se	3.7	0.3		0.7	0.5		13.2	0.8		8.0	0.3	
% Kelly	9.6	11.5	1.9	5.9	8.6	2.7	8.9	14.8	5.9	17.7	16.8	-0.9
se	3.2	0.2		3.1	0.4		4.4	0.5		2.8	0.2	
% Torrico	17.8	18.2	0.4	4.6	3.7	-0.9	6.4	11.3	4.9	9.8	10.1	0.3
se	4.7	0.2		2.5	0.3		3.5	0.5		4.5	0.2	
% Nava	16.4	17.7	1.3	3.5	1.1	-2.4	5.4	8.1	2.7	6.5	6.9	0.4
se	5.2	0.2		2.1	0.3		2.6	0.4		3.8	0.1	
% Schmier	4.0	4.0	0.0	1.8	3.5	1.7	3.4	3.8	0.4	5.0	4.6	-0.4
se	3.1	0.1		1.6	0.2		0.7	0.3		2.9	0.1	
Total	100.4	99.9	-0.5	92.1	100.1	8.0	82.2	100.0	17.8	101.6	99.9	-1.7

Precinct n = 4974, Number of Candidates = 7